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HEAT TRANSFER AT THE CLADDING–COOLING FLUID BOUNDARY

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Abstract—The example of the finite-difference method application for the heat transport calculations in the nuclear reactor channel is considered in this work. The system differential equations solution is presented in the comfortable way for the described case. These equations are sewn together by the boundary conditions on the surfaces between the system components separately considered. The particular simplification is obtained by the elimination of the coolant temperature in the finite-difference equations system. The coolant temperature is connected with the fuel and cladding temperature by the one-dimensional energy equation. In this way the calculations time economy is obtained. The presented method may be generalized for more complicated systems. Moreover this method assures the stability of the finite-difference equations system.

NOMENCLATURE

- c_p, heat capacity;
- q, heat flux;
- q_v, heat source per unit volume;
- r, radial coordinate in a cylindrical system;
- r_1, inner radius of the fuel pin;
- r_2, outer radius of the fuel pin, inner radius of the gas gap;
- r_3, outer radius of the gas gap, inner radius of the cladding;
- r_4, outer radius of the cladding, inner radius of the reactor channel;
- r_5, outer radius of the reactor channel;
- t, time;
- T, temperature;
- w, coolant velocity;
- z, axial coordinate in a cylindrical system.

Greek symbols
- α, convective heat transfer coefficient;
- λ, heat conductivity;
- ρ, density.

Subscripts
- c, coolant;
- cl, cladding;
- f, fuel pin;
- g, gas gap;
- s, saturation;
- cr, critical.

The gas gap conductivity is averaged as follows:

$$\bar{\lambda}_g(T_g) = \lambda_g\{\tfrac{1}{2}[T_g(z, r_2, t) + T_g(z, r_3, t)]\}.$$

1. INTRODUCTION

THE DISTRIBUTION of fuel temperature and also of other thermodynamic and hydraulic quantities in the reactor fuel element which are under consideration here are of great importance for the safety aspects of nuclear reactors. In particular, the former allows the critical heat flux to be predicted in the case of an accident of the main circulating pumps.

These reasons caused nuclear specialists to focus their attention on the above problem. We are citing some works in which the problem was tackled by means of the finite-difference method.

In [6] different heat exchanger systems are considered which are reduced to the hyperbolical partial differential sets of equations which are later solved by means of the finite-difference method.

The works of [7] and [8] contain the difference methods with the application to the thermal conductivity equation solution.

The work [9] describes a programme system for the phenomenon analysis in the fuel element during LOCA.

In the proposed paper the quasi-linear equation system describing the phenomenon has been changed into the quasi-linear differential equation system. Intentionally, has been indicated that the quasi-linear boundary condition of the third kind enables the elimination of the average coolant temperature from the obtained system of difference equations. It is of great practical importance due to complicated thermophysical relations in the reactor channel and allows the calculation time to be diminished.

2. DESCRIPTION OF THE PHYSICAL PROBLEM

An analysis of the problem of convective heat transfer to the cooling fluid is presented in this paper. The following assumptions have been introduced:

—there is axial symmetry of either the physical body or the physical properties' field in the fuel element and in the whole reactor channel as well;

—fuel element heat is conducted according to the Fourier equation;

—inside surface of the fuel is assumed to be an adiabatic wall and the heat is transferred from the

off-centre fuel surface through the gas gap toward the cladding;
—small heat capacity of the gas existing inside the gap allows treatment of the unsteady heat conduction within the gap as the steady state conduction phenomenon;
—heat conduction in the cladding is described by the Fourier equation;
—axial heat conduction in the channel is negligible;
—inlet coolant velocity is a time-dependent function and is given as a boundary condition;
—coolant temperature is assumed to be the mean value of the actual coolant temperature in radial direction;
—heat generation in the fuel element is a time- and axial-dependent function;
—steady state temperature distribution is assumed at the initial moment;
—coolant pressure is assumed to be constant at the whole length of the channel.

If we bore in mind the above assumptions then the unsteady state temperature distributions appearing in the particular areas of the considered system would be governed by energy conservation equations written for cylindrical coordinates as the following formulas:

in the fuel region

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\cdot\lambda_f(T_f)\cdot\frac{\partial T_f}{\partial r}\right]+q_v(z,t) = c_f(T_f)\cdot\rho_f(T_f)\cdot\frac{\partial T_f}{\partial t}; \quad (1)$$

in the gas gap region

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\cdot\bar{\lambda}_g(T_g)\cdot\frac{\partial T_g}{\partial r}\right]=0; \quad (2)$$

in the cladding region

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\lambda_{cl}(T_{cl})\cdot\frac{\partial T_{cl}}{\partial r}\right] = c_{cl}(T_{cl})\cdot\rho_{cl}(T_{cl})\cdot\frac{\partial T_{cl}}{\partial t}; \quad (3)$$

in the cooling fluid region

$$\frac{\mathrm{d}T_c}{\mathrm{d}t}=\frac{\partial T_c}{\partial t}+w(z,t)\cdot\frac{\partial T_c}{\partial z} = \frac{2\cdot r_4\cdot q(z,t)}{(r_5^2-r_4^2)\cdot\rho_c(T_c)\cdot c_{p_c}(T_c)}. \quad (4)$$

The boundary conditions are:

$$-\lambda_f(T_f)\cdot\left.\frac{\partial T_f}{\partial r_+}\right|_{r=r_1}=0; \quad (5)$$

$$T_f(z,r=r_2,t)=T_g(z,r=r_2,t); \quad (6)$$

$$-\lambda_f(T_f)\cdot\left.\frac{\partial T_f}{\partial r_-}\right|_{r=r_2} = -\bar{\lambda}_g(T_g)\cdot\left.\frac{\partial T_g}{\partial r_+}\right|_{r=r_2}; \quad (7)$$

$$T_g(z,r=r_3,t)=T_{cl}(z,r=r_3,t); \quad (8)$$

$$-\bar{\lambda}_g(T_g)\cdot\left.\frac{\partial T_g}{\partial r_-}\right|_{r=r_3} = -\lambda_{cl}(T_{cl})\cdot\left.\frac{\partial T_{cl}}{\partial r_+}\right|_{r=r_3}; \quad (9)$$

$$-\lambda_{cl}(T_{cl})\cdot\left.\frac{\partial T_{cl}}{\partial r_-}\right|_{r=r_4}=q(z,t) = \alpha\cdot[T_{cl}(z,r=r_4,t)-T_c(z,t)]. \quad (10)$$

Equations (1) and (3) are solved using the backward finite-difference method. The temperature distribution in the fuel region $r_1 < r < r_2$ is determined by the following formula:

$$T_f(z,r,t)=A_1\cdot T_f(z,r-\Delta r,t)+A_2\cdot T_f(z,r+\Delta r,t) + A_3\cdot T_f(z,r,t-\Delta t)+A_4. \quad (11)$$

If $r_1 = 0$ then

$$T_f(z,0,t)=A_1^0\cdot T_f(z,\Delta r,t) + A_2^0\cdot T_f(z,0,t-\Delta t)+A_3^0; \quad (12)$$

and if $r_1 > 0$ then

$$T_f(z,r_1,t)=(A_1+A_2)\cdot T_f(z,r_1+\Delta r,t) + A_3\cdot T_f(z,r_1,t-\Delta t)+A_4. \quad (13)$$

On the other hand for the fuel–gas gap boundary $r = r_2$ and we have

$$T_f(z,r_2,t)=T_g(z,r_2,t)=\frac{A_1+A_2}{1+\gamma_1\cdot A_2}\cdot T_f(z,r_2-\Delta r,t) + \frac{\gamma_1\cdot A_2}{1+\gamma_1\cdot A_2}\cdot T_g(z,r_3,t)+\frac{A_3}{1+\gamma_1\cdot A_2} \times T_f(z,r_2,t-\Delta t)+\frac{A_4}{1+\gamma_1\cdot A_2}. \quad (14)$$

For the steady state, $t = 0$, the fuel temperature distribution can be presented as

$$T_f(z,r,0)=B_1\cdot T_f(z,r-\Delta r,0) + B_2\cdot T_f(z,r+\Delta r,0)+B_3. \quad (15)$$

For the determination of the temperature distribution in the cladding region $r_3 < r < r_4$ equation (3) may be written in the following form:

$$T_{cl}(z,r,t)=C_1\cdot T_{cl}(z,r-\Delta r,t) + C_2\cdot T_{cl}(z,r+\Delta r,t) + C_3\cdot T_{cl}(z,r,t-\Delta t). \quad (16)$$

If $r = r_3$ then

$$T_{cl}(z,r_3,t)=T_g(z,r_3,t)=\frac{C_1+C_2}{1+\gamma_2\cdot C_1} \times T_{cl}(z,r_3+\Delta r,t)+\frac{\gamma_2\cdot C_1}{1+\gamma_2\cdot C_1} \times T_{cl}(z,r_2,t)+\frac{C_3}{1+\gamma_2\cdot C_1} \times T_{cl}(z,r_3,t-\Delta t). \quad (17)$$

The steady state temperature distribution in the cladding region is determined by the formula

$$T_{cl}(z,r,0) = D_1 \cdot T_{cl}(z,r-\Delta r,0) + D_2 \cdot T_{cl}(z,r+\Delta r,0). \quad (18)$$

Expressions (14) and (17) are based on the solution of equation (2) and on the boundary conditions (6) and (9). The formulas defining the coefficients: A_i, A_i^0, C_i, D_i and γ_i have been presented in the Appendix.

The representation of the conservation energy equation (4) solution for cooling medium consists of the characteristics, i.e. of the curves that fulfil the ordinary differential equations system,

$$\frac{dz}{dt} = w(z,t,T_c); \quad (19)$$

$$\frac{dT_c}{dt} = \beta \cdot [T_{cl}(z,r_4,t) - T_c(z,t)], \quad (20)$$

where

$$\beta = \frac{2 \cdot r_4 \cdot \alpha[T_c(z,t)]}{\rho_c[T_c(z,t)] \cdot c_{p_c}[T_c(z,t)] \cdot (r_5^2 - r_4^2)}. \quad (21)$$

Both equation (19) as the movement equation and equation (20) as the temperature one are referred to the coolant particles. The function $T_c(z,t)$ is known either for $t = 0$, as $T_c(z,0)$—the steady state distribution, or for $z = 0$, as the time-dependent function of the channel inlet temperature, $T_c(0,t)$. The characteristic equations are difficult to solve in the general case. The particular analysis for the cladding–coolant heat transfer is required to be made. Such analysis appoints an essentiality of this paper.

3. APPROXIMATION OF THE CHARACTERISTIC EQUATIONS SYSTEM SOLUTION

Several approximating assumptions have to be considered for the simplifying of the numerical calculations of the above-mentioned characteristic equations system. These assumptions run as follow. The variability range of t that we consider is chopped into equal-length parts Δt. The range of z is treated in the same way for obtaining Δz parts. As a result we can get the rectangular mesh in the (z,t) plane, as it is shown in Fig. 1. The characteristic equations (19) and (20) are considered then in the little rectangular computational area 1–2–3–4 situated within the mesh. This area is shown in Fig. 2.

3.1. *The fluid velocity determination within the computational rectangular area*

The first simplifying assumption we have made is that there is a constant fluid velocity inside the computational rectangle 1–2–3–4. This assumption is justified by the fact, that the area 1–2–3–4 may be chosen arbitrarily small depending upon the chopping performed. It can therefore be assumed with negligible error that the fluid particle is at constant velocity in the considered area. From this assumption it follows that the particle trajectory imitated on the (z,t) plane becomes the segment 5–4 of the straight line. This line runs from the point 4 toward either side 1–2 or side 1–3 of the considered rectangle as it has shown in Figs. 3 and 4. All particle velocities at the rectangle corners are known except the velocity at point number 4. This one will be determined later using the continuity equation and eventually the other relations. The typical case is that the velocities at the points 1, 2 and 3 do not differ too much from each other in view of the small size of the rectangle and the velocity continuity. In further considerations we shall admit little differences between the fluid velocity at these points.

If it is assumed that

$$w \gg 0 \quad (22)$$

then the particle trajectory must intersect either segment 1–2 or segment 1–3, as shown in Figs. 3 and 4. These velocities may be interpolated linearly in the segments 1–2 and 1–3. Subsequently we can create a new point (number 5) on one of these segments such that the particle trajectory imitated on the (z,t) coordinates includes points 5 and 4. The segment obtained is so inclined as a result of the linear velocity interpolation. This straight line is accepted as an approximation of the characteristic projection within the rectangle 1–2–3–4. All of the computations are necessary to determine the point 5.

Point 5 can be determined from the condition

$$w_5 \cdot p \cdot \Delta t = q \cdot \Delta z \quad (23)$$

where p and q are the point 5 coordinates.

If we used the linear interpolation for the segment 1–3—in this case $p = 1$—then we should obtain

$$q = \frac{w_3}{w_3 + \dfrac{\Delta z}{\Delta t} - w_1}. \quad (24)$$

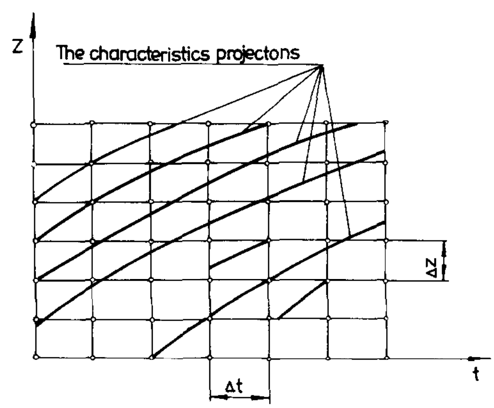

FIG. 1. The rectangular mesh in the (z,t) plane and the characteristics projected on this plane.

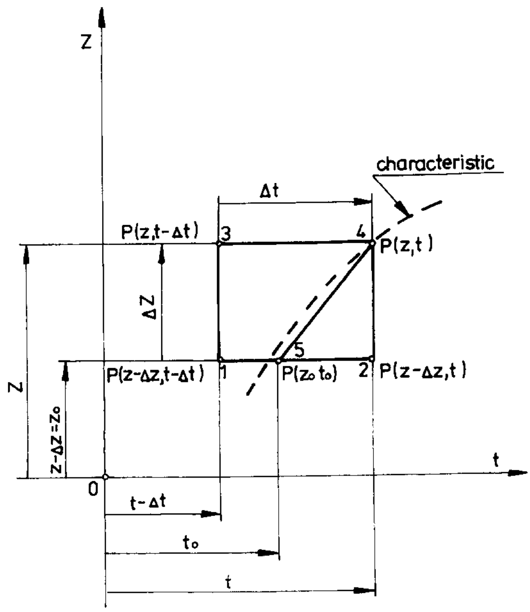

FIG. 2. The little calculation rectangle.

If $q \leqslant 1$ then these coordinates determine the point 5 position within the segment 1–3. This is shown in Fig. 4.

On the other hand if $q > 1$ then the interpolation equation has to be set for the segment 1–2. In this case $q = 1$ and therefore we obtain

$$p = \frac{w_2}{2(w_2 - w_1)} \cdot (1 - \sqrt{(1 - \xi)}) \tag{25}$$

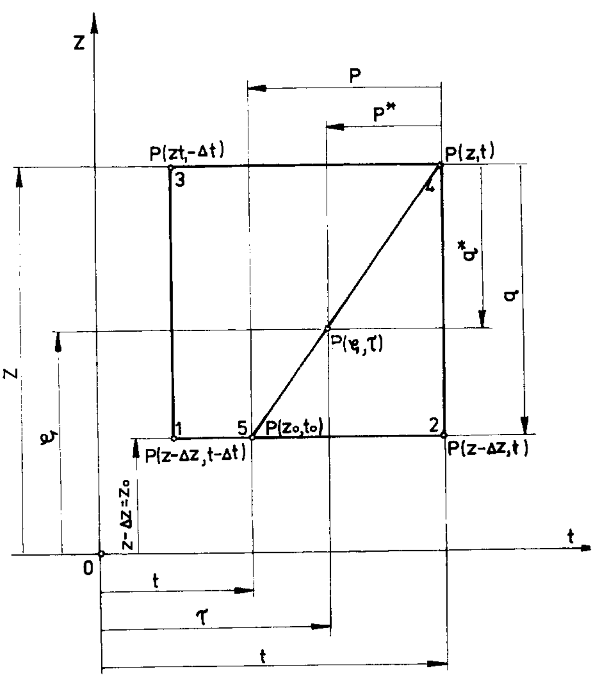

FIG. 3. The calculation rectangle for $w > \Delta z/\Delta t$.

where

$$\xi = \frac{4(w_2 - w_1) \cdot \dfrac{\Delta z}{\Delta t}}{w_2^2}. \tag{26}$$

For $|\xi| < 1$ we can apply formula [1]

$$1 - \sqrt{(1 - \xi)} = \frac{1}{2} \cdot \xi \cdot \left[1 + \sum_{i=1}^{\infty} \frac{(2 \cdot i - 1)!!}{2^i \cdot i!} \cdot \xi^i\right] \tag{27}$$

and we will obtain the sought formula for p

$$p = \frac{\dfrac{\Delta z}{\Delta t}}{w_2}\left[1 + \sum_{i=1}^{\infty} \frac{(2 \cdot i - 1)!!}{2^i \cdot i!} \cdot \xi^i\right]. \tag{28}$$

In this way we have appointed the position of point 5 so that this point is situated within the segment 1–2. Subsequently, based on the linear interpolation, we calculate the cooling fluid temperature T_c at point 5 and determine the values of ρ_c, c_{p_c} and α according to this temperature. These coefficients appear in formula (21), which defines the coefficient β. Furthermore we assume these parameters to be constant within the segment 5–4 just as we have done for velocity w. In consequence this leads to the following assumption

$$\beta = \beta[T_c(z_0, t_0)]. \tag{29}$$

The characteristic equations are subsequently integrated in the next subsection.

3.2. *The integration of the characteristic equations in the computational rectangle*

3.2.1. *The transformation of the coordinates system.* The point (z, t), arbitrarily fixed, is the computational rectangle corner, which we denote as point 4 (Figs. 2 and 4). We denote the current coordinates as (ζ, τ). The new coordinates p^* and q^*—conveniently computable in the considered rectangle—are defined by formulas

$$\zeta = z - q^* \cdot \Delta z, \quad \tau = t - p^* \cdot \Delta t. \tag{30}$$

The functions appearing in the continuation of this work have been denoted according to the relationship

$$T(\tau) = T^{**}[\zeta_{ch}(\tau), \tau] = T^{**}(p^*, q^*)\Big|_{\substack{p^* = (t - \tau)/\Delta t \\ q^* = [z - \zeta_{ch}(\tau)]/\Delta z}} \tag{31}$$

where $\zeta_{ch}(\tau)$ indicates the function defining the characteristic projection at the plane (ζ, τ).

3.2.2. *The solution of the characteristic equations.* As an approximate solution of the characteristic equation

$$\frac{d\zeta_{ch}(\tau)}{d\tau} = w[\zeta_{ch}(\tau), \tau, T_c]; \tag{32}$$

the straight line equation in the computational rectangle is assumed to be

$$q_{ch}^*(p^*) = \frac{q}{p} p^*. \tag{33}$$

The solution of the second characteristic equation,

$$\frac{dT_c}{dt} = \beta\cdot(T_{cl} - T_c), \tag{34}$$

can be written as [2]

$$T_c^{**}\left(\frac{q}{p}\cdot p^*, p^*\right) = T_c^{**}(q,p)\cdot\exp\left[k(p-p^*)\right]$$
$$+ k\cdot\exp(-k\cdot p^*)\cdot\int_p^{p^*} T_{cl}^{**}\left(\frac{q}{p}\cdot\bar{p},\bar{p}\right)\cdot\exp(k\cdot\bar{p})\,d\bar{p} \tag{35}$$

where

$$k = -\beta\cdot\Delta t. \tag{36}$$

The numerical values of the functions T_{cl}^{**} and T_c^{**} may be calculated using the linear interpolation. For just that reason we assume, that T_{cl}^{**} can be expressed as

$$T_{cl}^{**}(q^*,p^*) = q^*\cdot p^*\cdot T_{cl_1} + q^*(1-p^*)\cdot T_{cl_2}$$
$$+ (1-q^*)\cdot p^* T_{cl_3} + (1-q^*)\cdot(1-p^*)\cdot T_{cl_4}. \tag{37}$$

Furthermore, upon noting that

$$\left\{\begin{aligned} a &= (T_{cl_1} - T_{cl_2} - T_{cl_3} + T_{cl_4})\cdot\frac{q}{p} \\ b &= \frac{q}{p}\cdot(T_{cl_2} - T_{cl_4}) + T_{cl_3} - T_{cl_4} \\ c &= b - 2\cdot\frac{a}{k} \\ d &= T_{cl_4} - \frac{c}{k} \end{aligned}\right. \tag{38}$$

we find the solution in the following form

$$T_c^{**}(0,0) = T_c^{**}(q,p)$$
$$-\left[(a\cdot p + c)\cdot p + d)\right]\cdot\exp(p\cdot k) + \mathrm{d} \tag{39}$$

or

$$T_c(z,t) = T_c(z_0,t_0)$$
$$-\left[(a\cdot p + c)\cdot p + d)\right]\cdot\exp(-p\cdot\beta\cdot\Delta t) + d. \tag{40}$$

This way the temperature of the cooling fluid at the point (z,t) is calculated. This point is denoted as the number 4 (Figs. 2 and 4). It must be emphasized that the temperature $T_c^{**}(q,p) = T_c(z_0,t_0)$ has been calculated as a result of the linear interpolation of the known values of T_{c_1}, T_{c_2} and T_{c_3}. The value of the coefficient β in the little rectangle 1–2–3–4 is constant, but only in the first approximation. In the first step of the iteration we assume that

$$T_{cl}(z,r_4,t) = T_{cl}(z,r_4,t-\Delta t). \tag{41}$$

After the computation of the temperature $T_c(z,t)$ the fluid velocity at the point (z,t) can be obtained based on the continuity equation

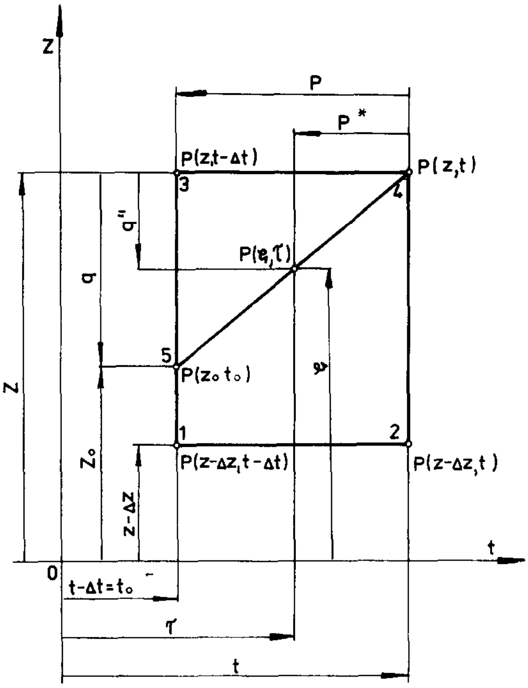

FIG. 4. The calculation rectangle for $w \leqslant \Delta z/\Delta t$.

$$w(z,t) = \frac{\rho_c[T_c(0,t)]\cdot w(0,t) - \displaystyle\int_0^z \frac{\partial\rho_c}{\partial t}\bigg|_{\substack{t=t\\ z=\zeta}}\cdot d\zeta}{\rho_c[T_c(z,t)]}. \tag{42}$$

The integral in (42) can be evaluated approximately because $\rho_c\,(z,t)$ for $0 \leqslant \tau \leqslant t$, $0 \leqslant \zeta \leqslant z$ can be assumed known. In addition, $w(0,t)$ is the inlet velocity assumed to be a given function of time and represented as

$$w(0,t) = w_0\cdot w_t(t) \tag{43}$$

where

$w_0 = w(0,0)$ is the inlet fluid velocity for $t = 0$, and
$w_t(t)$ is the non-dimensional rate of the fluid velocity as a function of the time.

Therefore $w(z,0)$ for $z > 0$ and $w(0,t)$ for $t > 0$ can be assumed known.

Returning to the scheme of differences, one obtains a quasi-linear formula for the temperature of the wall:

$$T_{cl}(z,r_4,t) = \frac{C_1 + C_2}{1 + \gamma_3\cdot C_2}\cdot T_{cl}(z,r_4-\Delta r,t)$$
$$+ \frac{\gamma_3\cdot C_2}{1+\gamma_3\cdot C_2}\cdot T_c(z,t)$$
$$+ \frac{C_3}{1+\gamma_3\cdot C_2}\cdot T_{cl}(z,r_4,t-\Delta t) \tag{44}$$

where

$$\gamma_3 = \frac{2\alpha[T_c(z,t)]\cdot\Delta r}{\lambda_{cl}[T_{cl}(z,r_4,t)]} \tag{45}$$

The system of difference equations for fuel, gas gap, cladding and coolant is solved by iterations. The right hand sides contain the unknowns in the form of weakly changing coefficients or in the non-linear form as for the coolant temperature. Iterations are terminated when the unknowns change little. The computation has shown the convergence of those iterations. Later we found that the elimination of the coolant temperature $T_c(z,t)$ from formula (44), with the help of (40), leads to quasi-linear equations which fulfil sufficient conditions for the convergence of the iterations. To do this substitution we write (40) as

$$\begin{aligned} T_c(z,t) &= \delta_1\cdot T_{cl}(z-\Delta z,r_4,t-\Delta t)\\ &\quad+\delta_2\cdot T_{cl}(z-\Delta z,r_4,t)\\ &\quad+\delta_3\cdot T_{cl}(z,r_4,t-\Delta t)\\ &\quad+\delta_4\cdot T_{cl}(z,r_4.t)+\delta_5 \end{aligned} \tag{46}$$

where the formulas for coefficients δ_i are given in the Appendix. Next, the relation (46) is substituted into formula (44). The new equation obtained in this way is solved for the temperature $T_{cl}(z,r_4,t)$ and one obtains

$$\begin{aligned} T_{cl}(z,r_4,t) &= \frac{C_1+C_2}{1+\gamma_3\cdot C_2(1-\delta_4)}\\ &\quad\times T_{cl}(z,r_4-\Delta r,t)\\ &\quad+Q\cdot\frac{1+\gamma_3\cdot C_2}{1+\gamma_3\cdot C_2\cdot(1-\delta_4)} \end{aligned} \tag{47}$$

where

$$\begin{aligned} Q &= \frac{\gamma_3\cdot C_2}{1+\gamma_3\cdot C_2}\cdot[\delta_1\cdot T_{cl}(z-\Delta z,r_4,t-\Delta t)\\ &\quad+\delta_2\cdot T_{cl}(z-\Delta z,r_4,t)\\ &\quad+\delta_3\cdot T_{cl}(z,r_4,t-\Delta t)+\delta_5]\\ &\quad+\frac{C_3}{1+\gamma_3\cdot C_2}\cdot T_{cl}(z,r_4,t-\Delta t). \end{aligned} \tag{48}$$

The convergence of the iteration process requires that the condition

$$\left|\frac{C_1+C_2}{1+\gamma_2\cdot C_2\cdot(1-\delta_4)}\right|<1 \tag{49}$$

be assured. This is the consequence of the convergence criterion [3]. Condition (49) is accomplished for $\Delta t>0$. Also the finite-difference equations system (11), (12), (13), (14) and (17) provides the convergency criterion [3]:

$$\begin{gathered} |A_1|+|A_2|<1\\ |A_1^0|<1\\ |D_1|+|D_2|<1\\ \left|\frac{A_1+A_2}{1+\gamma_1\cdot A_2}\right|+\left|\frac{\gamma_1\cdot A_2}{1+\gamma_1\cdot A_2}\right|<1\\ \left|\frac{C_1+C_2}{1+\gamma_2\cdot C_1}\right|+\left|\frac{\gamma_2\cdot C_1}{1+\gamma_2\cdot C_1}\right|<1. \end{gathered} \tag{50}$$

Condition (50) is satisfied for $\Delta t>0$, as well. The singularity and quasi-singularity of the equations system (11), (12), (13), (14), (17) and (47) are eliminated in this way. Accidental errors are avoided too. The feature of the algorithm, presented above, is that it is absolutely stable.

4. NUMERICAL EXAMPLE

The above-described method is used to estimate the failure consequences in the PW-1 pressurized water loop. This loop simulates a sub-assembly of the WWER-1000 power reactor. As the above-mentioned failure we consider the shut-down of the principal circuit pump. In order to simplify the task the analysis is performed for only one cell. We would like to note that in this case the equivalent annulus model is acceptable. The calculations are performed under the following assumptions:

(1) The thermal critical flux is calculated using the Smolin–Poliakov's correlation [4].

(2) The safety rods drop-down according to the characteristic presented in [5].

(3) The inlet velocity distribution is determined by the formula

$$w(z=0,t)=\frac{w_0}{1+2.2\cdot t}. \tag{51}$$

(4) The fuel–gas gap–cladding element data are the same in the preliminary project.

The results of these calculations allow us to answer the principal question: How long can the period of time between the beginning of the pump failure and the safety rods drop realization moment be in order to avoid the appearance of the boiling crisis? The answer, about 4 seconds, is given in the Figs. 5 and 6. The above result must be considered only as a preliminary one. Numerical calculations should be performed again when the new data concerning the whole loop circuit construction and the operation conditions are given.

5. REMARKS

The algorithm of the calculations, presented in this paper, makes it possible to solve the unsteady heat transfer problems for the nuclear reactor. The transient perturbations such as the coast-down of the flow of the cooling fluid or the change of the fuel heat generation are postulated. The physical properties of the reactor components are the functions of the

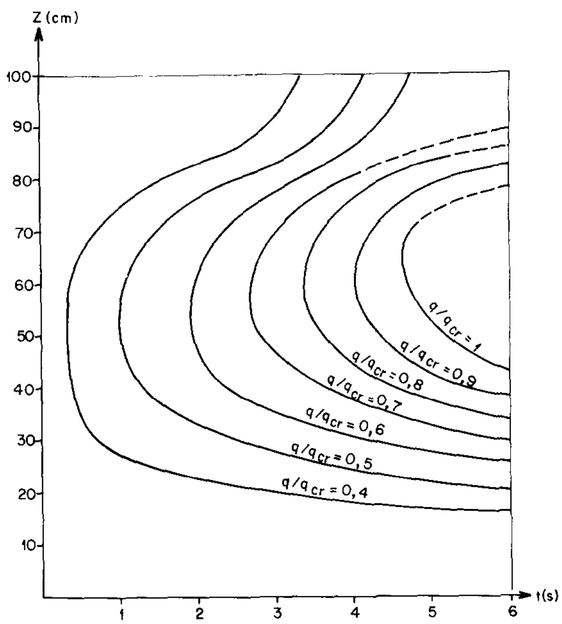

FIG. 5. The ratio of the heat flux on the cladding surface to the critical heat flux on the (z, t) plane.

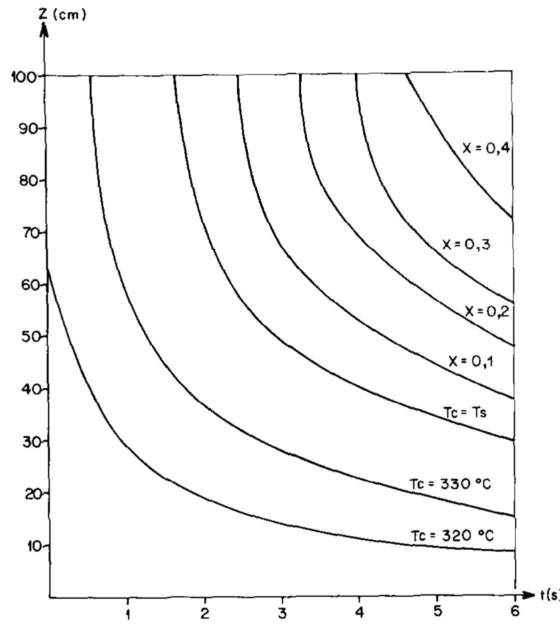

FIG. 6. The coolant temperature and the steam quality on the (z, t) plane.

temperature. These properties are tabulated. The functions are approximated by the Hermite interpolation polynomial.

The algorithm presented in this paper is designed for the unsteady thermohydrodynamic processes calculations and—with some modifications—was applied in the fast reactor dynamics analysis for the dissociating gas N_2O_4 cooled reactor.

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APPENDIX

The coefficients A_i are defined as follows:

$$A_1 = \frac{\left\{\lambda_f[T_f(z,r,t)] \cdot \left(1 - \frac{\Delta r}{2 \cdot r}\right) - \frac{\lambda_f[T_f(z,r+\Delta r,t)] - \lambda_f[T_f(z,r-\Delta r,t)]}{4}\right\}\Delta t}{2 \cdot \lambda_f[T_f(z,r,t)] \cdot \Delta t + \rho_f[T_f(z,r,t)] \cdot c_f[T_f(z,r,t)] \cdot (\Delta r)^2};$$

$$A_2 = \frac{\left\{\lambda_f[T_f(z,r,t)] \cdot \left(1 + \frac{\Delta r}{2 \cdot r}\right) + \frac{\lambda_f[T_f(z,r+\Delta r,t)] - \lambda_f[T_f(z,r-\Delta r,t)]}{4}\right\} \cdot \Delta t}{2 \cdot \lambda_f[T_f(z,r,t)] \cdot t + \rho_f[T_f(z,r,t)] \cdot c_f[T_f(z,r,t)] \cdot (\Delta r)^2};$$

$$A_3 = \frac{\rho_f[T_f(z,r,t)] \cdot c_f[T_f(z,r,t)] \cdot (\Delta r)^2}{2 \cdot \lambda_f[T_f(z,r,t)] \cdot \Delta t + \rho_f[T_f(z,r,t)] \cdot c_f[T_f(z,r,t)] \cdot (\Delta r)^2};$$

$$A_4 = \frac{\Delta t \cdot (\Delta r)^2 \cdot q_v(z,t)}{2 \cdot \lambda_f[T_f(z,r,t)] \cdot \Delta t + \rho_f[T_f(z,r,t)] \cdot c_f[T_f(z,r,t)] \cdot (\Delta r)^2}.$$

The coefficients A_i^0 are determined by the following expressions:

$$A_1^0 = \frac{4 \cdot \lambda_f[T_f(z,0,t)] \cdot \Delta t}{4 \cdot \lambda_f[T_f(z,0,t)] \cdot \Delta t + \rho_f[T_f(z,0,t)] \cdot c_f[T_f(z,0,t)] \cdot (\Delta r)^2};$$

$$A_2^0 = \frac{\rho_f[T_f(z,0,t)] \cdot c_f[T_f(z,0,t)] \cdot (\Delta r)^2}{4 \cdot \lambda_f[T_f(z,0,t)] \cdot \Delta t + \rho_f[T_f(z,0,t)] \cdot c_f[T_f(z,0,t)] \cdot (\Delta r)^2};$$

$$A_3^0 = \frac{\Delta t \cdot (\Delta r)^2 \cdot q_v(z,t)}{4 \cdot \lambda_f[T_f(z,0;t)] \cdot \Delta t + \rho_f[T_f(z,0,t)] \cdot c_f[T_f(z,0,t)] \cdot (\Delta r)^2}.$$

Next, we assume that

$$B_1 = \frac{\lambda_f[T_f(z,r,0)] \cdot \left(1 - \dfrac{\Delta r}{2 \cdot r}\right) - \dfrac{\lambda_f[T_f(z,r+\Delta r,0)] - \lambda_f[T_f(z,r-\Delta r,0)]}{4}}{2 \cdot \lambda_f[T_f(z,r,0)]};$$

$$B_2 = \frac{\lambda_f[T_f(z,r,0)] \cdot \left(1 + \dfrac{\Delta r}{2 \cdot r}\right) + \dfrac{\lambda_f[T_f(z,r+\Delta r,0)] - \lambda_f[T_f(z,r-\Delta r,0)]}{4}}{2 \cdot \lambda_f[T_f(z,r,0)]};$$

$$B_3 = \frac{(\Delta r)^2 \cdot q_v(z,0)}{2 \cdot \lambda_f[T_f(z,r,0)]}$$

and

$$C_1 = \frac{\Delta t \left(\lambda_{cl}[T_{cl}(z,r,t)] \cdot \left(1 + \dfrac{\Delta r}{2 \cdot r}\right) + \dfrac{1}{4}\{\lambda_{cl}[T_{cl}(z,r+\Delta r,t)] - \lambda_{cl}[T_{cl}(z,r-\Delta r,t)]\} \right)}{2 \cdot \lambda_{cl}[T_{cl}(z,r,t)] \cdot \Delta t + \rho_{cl}[T_{cl}(z,r,t)] \cdot c_{cl}[T_{cl}(z,r,t)] \cdot (\Delta r)^2};$$

$$C_2 = \frac{\Delta t \left(\lambda_{cl}[T_{cl}(z,r,t)] \cdot \left(1 - \dfrac{\Delta r}{2 \cdot r}\right) - \dfrac{1}{4}\{\lambda_{cl}[T_{cl}(z,r+\Delta r,t)] - \lambda_{cl}[T_{cl}(z,r-\Delta r,t)]\} \right)}{2 \cdot \lambda_{cl}[T_{cl}(z,r,t)] \cdot \Delta t + \rho_{cl}[T_{cl}(z,r,t)] \cdot c_{cl}[T_{cl}(z,r,t)] \cdot (\Delta r)^2};$$

$$C_3 = \frac{\rho_{cl}[T_{cl}(z,r,t)] \cdot c_{cl}[T_{cl}(z,r,t) \cdot (\Delta r)^2}{2 \cdot \lambda_{cl}[T_{cl}(z,r,t)] \cdot \Delta t + \rho_{cl}[T_{cl}(z,r,t)] \cdot c_{cl}[T_{cl}(z,r,t)] \cdot (\Delta r)^2};$$

$$D_1 = \frac{\lambda_{cl}[T_{cl}(z,r,0)] \cdot \left(1 - \dfrac{\Delta r}{2 \cdot r}\right) - \dfrac{1}{4}\{\lambda_{cl}[T_{cl}(z,r+\Delta r,0)] - \lambda_{cl}[T_{cl}(z,r-\Delta r,0)]\}}{2 \cdot \lambda_{cl}[T_{cl}(z,r,0)]};$$

$$D_2 = \frac{\lambda_{cl}[T_{cl}(z,r,0)] \cdot \left(1 + \dfrac{\Delta r}{2 \cdot r}\right) + \dfrac{1}{4}\{\lambda_{cl}[T_{cl}(z,r+\Delta r,0)] - \lambda_{cl}[T_{cl}(z,r-\Delta r,0)]\}}{2 \cdot \lambda_{cl}[T_{cl}(z,r,0)]}.$$

The coefficients γ_i are defined as follows:

$$\gamma_1 = \frac{\Delta r \cdot \bar{\lambda}_g}{r_2 \cdot \ln\left(\dfrac{r_3}{r_2}\right) \cdot \lambda_f[T_f(z,r_2,t)]};$$

$$\gamma_2 = \frac{2 \cdot \Delta r \cdot \bar{\lambda}_g}{r_3 \cdot \ln\left(\dfrac{r_2}{r_3}\right) \cdot \lambda_{cl}[T_{cl}(z,r_3,0)]}.$$

The coefficients δ_i are defined as follows:

$$\delta_1 = \frac{2 \cdot q}{p \cdot \beta^2 \cdot (\Delta t)^2}[1 - \exp(-p \cdot \beta \cdot \Delta t)] - \frac{q(2 + p \cdot \beta \cdot \Delta t)}{\beta \cdot \Delta t} \cdot \exp(-p \cdot \beta \cdot \Delta t);$$

$$\delta_2 = \frac{q(\beta \cdot \Delta t - 2)}{p \cdot \beta^2 \cdot (\Delta t)^2}[1 - \exp(-p \cdot \beta \cdot \Delta t)] - \frac{q[\beta \cdot \Delta t(1-p) - 2]}{\beta \cdot \Delta t} \cdot \exp(-p \cdot \beta \cdot \Delta t);$$

$$\delta_3 = \frac{p \cdot \beta \cdot \Delta t - 2 \cdot q}{p \cdot \beta^2 \cdot (\Delta t)^2}[1 - \exp(-p \cdot \beta \cdot \Delta t)] - \frac{p \cdot \beta \cdot \Delta t(1-q) - 2q}{\beta \cdot \Delta t} \cdot \exp(-p \cdot \beta \cdot \Delta t);$$

$$\delta_4 = \frac{p \cdot \beta \cdot \Delta t(\beta \cdot \Delta t - 1) - q(\beta \cdot \Delta t - 2)}{p \cdot \beta^2 \cdot (\Delta t)^2}[1 - \exp(-p \cdot \beta \cdot \Delta t)] - \frac{\beta \cdot \Delta t(q \cdot p - q - p) + 2q}{\beta \cdot \Delta t} \cdot \exp(-p \cdot \beta \cdot \Delta t);$$

$$\delta_5 = T_c(z_0, t_0) \cdot \exp(-p \cdot \beta \cdot \Delta t).$$

L'ECHANGE DE LA CHALEUR ENTRE LA GAINE ET LE FLUID REFRIGERANT

Résume—On a examiné l'exemple d'utilisation de la méthode aux différences finies aux calculs d'échange de la chaleur dans un canal d'une pile nucléaire. On a montré un mode de solution du système d'équations différentielles, très commode pour le cas consideré. Les équations sont couplées les unes aux autres par les conditions aux limites entre les zones distincts du système consideré. On a gagné une simplification considérable—conduisante à l'économie du temps de calculs—par l'élimination, dans le schème aux différences, de la température du fluide réfrigérant, couplée avec la température du combustible et de la gaine par intermédiaire de l'équation unidimensionelle de l'énergie. C'est une équation quasilinéaire valable dans la zone du canal avec le fluide réfrigérant. La méthode peut-être généralisée aux systèmes, plus complexes y compris l'échange de la chaleur par la conduction et la convection. Elle garantie aussi la stabilité du système d'équations aux différences, qui rapprochent l'écoulement du processus donné.

NUMERISCHE BERECHNUNG DER WÄRMEÜBERTRAGUNG IN DEN BRENNELEMENTEN EINES KERNREAKTORS

Zusammenfassung—In der vorliegenden Arbeit ist ein Differenzverfahren angegeben, welches die Bestimmung des Wärmetransport in den Kernreaktorskanälen gestattet. Das Verfahren ist besonders zur Lösung der Systeme von Differentialgleichungen geeignet. An Rändern des Integrationsintervalls entsprechender Differentialgleichungen sind die zugeschriebenen Randwerte berücksichtigt worden.

Es lässt sich die Berechnung beschleunigen, indem man die Temperatur des Kühlmittels, die mit der Temperatur des Brennelementes und dessen Ummantelung verbunden ist, eliminiert.

Das Berechnungsverfahren kann auch auf komplizierte Fälle übertragen werden. Es ist auch anwendbar, wenn die Wärmeübertragung aus der Wärmeleitung und der Konvektion zusammengesetzt ist. Unabhängig davon versichert das angegebene Verfahren, dass das Berechnungschema stabil wird.

ТЕПЛОПЕРЕНОС НА ГРАНИЦЕ РАЗДЕЛА ОБОЛОЧКА-ОХЛАЖДАЮЩАЯ ЖИДКОСТЬ

Аннотация — Рассмотрен пример использования конечно-разностного метода для расчета переноса тепла в канале ядерного реактора. Решение системы дифференциальных уравнений представлено в удобной форме. Уравнения решаются совместно с граничными условиями на поверхностях между отдельными областями рассматриваемой системы. Существенное упрощение получено за счет исключения в конечно-разностной схеме температуры теплоносителя, связанной с температурой топливного элемента и оболочки одномерным уравнением энергии. Тем самым получена экономия времени счета. Метод обеспечивает устойчивость конечно-разностной схемы. Представленный метод можно обобщить на более сложные системы.